ROY

Analysis of a
100-Foot Concrete Arch

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ANALYSIS

OF A

100-FT. CONCRETE ARCH

BY

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THESIS

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This is to certify that the thesis prepared under the immediate supervision of Instructor L. A. Waterbury by

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entitled ANALYSIS OF A 100-FOOT CONCRETE ARCH

is approved by me as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering

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OBJECT.

The object of this thesis is to determine analytically the stres-stresses, due to live load, dead load and temperature changes, in the 100 ft. plain concrete arch in the west approach to the Thebes bridge, Thebes, Illinois.

DIMENSIONS OF ARCH.

The arch to be analysed is the largest in the west approach to the railroad bridge over the Mississippi river at Thebes, Illinois. The arch ring has a radial depth of 4 1/2 feet at the crown and 11 feet at the haunches. The curve of the intrados has a radius of 50 feet; the curve of the extrados, a radius of 62 feet 1 5/8 inches, from the crown to a point 40 degrees 1 minute from the vertical where it becomes a tangent. The clear span of the arch is 100 feet; the rise, 50 feet.

Fig. 1 shows the form and dimensions of the arch.

LOADING.

The arch was designed for a vertical live load of 6200 pounds per lineal foot of track. The earth filling was assumed to weigh 105 pounds per cubic foot, and concrete, 150 pounds per cubic foot.

These loads and assumptions will be used in this analysis.

METHOD OF ANALYSIS.

The analysis was made using the tables given in "Calculation of Stresses and Practical Design of Structures of Steel Concrete" by Walter W. Colpitts, Assistant Chief Engineer, K. C. M. & O. Ry. The tables are based upon the elastic theory of the parabolic arch as presented in the "Treatise On Arches" by M. A. Howe and in Johnson's "Modern Framed Structures". All equations quoted are taken from these treatises.

According to the theory of parabolic arches, if a vertical load, w, should be placed upon an arch at any point, three conditions will be satisfied, namely,-

(1) There will be no change of span.

(2) The algebraic sum of the changes of inclination between the abutments will be zero.

(3) The algebraic sum of the deflections between the abutments will be zero.

These conditions will also be satisfied if any number of such loads were applied at the same time.

The position of the equilibrium polygon for this vertical load, w, or any combination of vertical loads, can be determined and the stresses produced thereby calculated, but this would necessitate the drawing of a number of such equilibrium polygons for the various loadings.

If the effect of each load separately is determined and then the stresses caused by the different loads of any combination which produces a maximum at any particular point are summed up, the process is much simplified. It is for this simplification that Mr. Colpitts has arranged the tables given on pages 15-17.

Referring to the parabola of fig. II,

if k = rise,

c = 1/2 span,

x = abscissa of any point, P, on the parabola,

y = ordinate of any point, P, on the parabola,

then $y = k(1 - \frac{x}{c})$ by means of which the location of the point, P, can be determined.

Referring to Fig. 3, the equilibrium polygon for a single ver-

tical load, w, is represented by the two straight lines, ln and ne, intersecting at the point n in the vertical line through the center of gravity of w. y_0, y_1, and y_2 are the ordinates from the springing lines which locate the equilibrium polygon.

If b = horizontal distance from w to center of span, also = nc,

c = 1/2 span,

k = rise,

n = ratio of b to c,

then $y_0 = 6/5\,k$ - (1)

$y_1 = 2/15\,\frac{1+5n}{1+n}.k$ - (2)

$y_2 = 2/15.\frac{1-5n}{1-n}.k$ - (3)

y_0 is independent of the span and therefore constant for all positions of w. The locus of y_0 is mno which is 6/5 of the rise above and parallel to AB. The vertical components of the abutment reactions

$P_1 = \frac{(2-n)(1+n)^2}{4}.w$ - (4)

$P_2 = \frac{(2+n)(1-n)^2}{4}.w$ - (5)

The horizontal component

$H = 15/32\,(1-n^2)^2\,c/k.w$ - (6)

Table 1 gives the values of y_0, y_1, y_2, P_1, P_2, and H for w in different positions. By means of this table the bending moment at any point, as at L, Fig. 3, with w in any position may be calculated thus;-

The ordinates, y_0, y_1, and y_2, may be taken from the table and the equilibrium polygon drawn. Then, by measuring the intercept, Lf = z, between the linear arch and the equilibrium polygon and multiplying it by H, the bending moment, M = Hz, is obtained, being positive or negative according as the equilibrium polygon falls above or below L.

Referring to Fig. 4, the equilibrium polygon for a single hori-

zontal load, H, is represented by the two straight lines, LG and GN, intersecting at the point G in the horizontal line thru the center of gravity of H.

If x_o= abscissa of G from the center of span (G will always be on the side of the center opposite that to which H is applied).

x_1 and x_2= abscissae from the springing line which locate the equilibrium polygon,

b = horizontal distance from H to center of span, also = nc,

c = 1/2 span,

k = rise,

n = ratio of b to c,

then $x_o = 2n^3c$ -(7)

$x_1 = c/3(1 + \frac{4n^2}{1 - n})$ -(8)

$x_2 = c/3(1 + \frac{4n^2}{1 + n})$ -(9)

The vertical components of the abutment reactions

$P_1 = - P_2 = 3/8\ Hk/c\ (1 - n^2)^2$ - - - - - - - - - - - - - - - - - - - (10)

The horizontal components

$H_1 = H/3(1 + n/2\ (5 - 3n^2))$ - - - - - - - - - - - - - - - - - - - (11)

$H_2 = H - H_1$ -(12)

Table 2 gives the values of x_o, x_1, x_2, P, H_1 and H_2 for H in different positions. By means of this table the bending moment at a any point, as at c, Fig. 4 with H in any position may be calculated thus;-

x_o, x_1, and x_2 may be taken from the table and the equilibrium polygon drawn. Then, by measuring the intercept, CD = r, between the linear arch and the equilibrium polygon and multiplying it by P,the bending moment, M = Pr, is obtained, being positive or negative according as the equilibrium falls to the right or left of c.

The discussion thus far has been for a single load. For any

combination of loads a simple extension is necessary. The effects of of each load separately are added.

Thus, to determine the stresses in an arch ring due to any particular combination of dead and live loads, the arch is divided into a certain number of equal panels and both horizontal and vertical loads are considered to be concentrated at the points of subdivision.

For an arch divided into ten equal panels Table 3 gives the coefficients which, when multiplied by 1/2 span and by the vertical load, give the bending moment in inch-pounds at any panel point due to that load.

Table 4 gives the coefficients which, when multiplied by the rise and by the horizontal load, give the bending moment in inch-pounds at any panel point due to that load.

No further explanation of these tables need here be given as the analysis to follow illustrates their use. W and H may represent either the live or dead loads at the point considered.

EQUATION OF CENTER LINE.

Before working out any stresses, the arch was investigated by means of drawings made to scale to see if the medial line of the arch rib is a parabola. This was found to be nearly enough true for the analysis.

The arch was further investigated to determine whether $A = E\theta \cos \phi$ = constant; E being the coefficient of elasticity of concrete and thus constant for all sections, θ, the moment of inertia of any section and ϕ, the angle from the vertical through the crown to the section taken. Fig. 5 and table 5 show the location of the sections and the results deduced. It is to be noticed that the relation $A = E\theta \cos \phi^{3.26\pm.31}$ = constant was calculated by the theory of least squares. It would seem that this power of cos ϕ is caused by the

excess of material in the arch.

DETAILS OF ANALYSIS.

Moments due to vertical live loads.

The arch was divided into ten equal panels of ten feet each. Since the loads were considered concentrated at the panel points each panel load = 7750 pounds = w. c = 1/2 span = 50 feet. By use of table 3 the moments at the abutment were found as follows:

Max. positive M = +4.068 x 50 x 7750 = +157630 inch pounds.

Max. negative M = -3.864 x 50 x 7750 = -149730 inch pounds.

The moments at each of the other panel points were determined in the same manner. The results are as follows:

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Max. positive M	+157630	+47890	+74540	+94860	+72540	+58590
Max. negative M	-149730	-39990	-68350	-87420	-64630	-50220

Table 3 and other tables show that the moments are the same for each half of the arch.

Moments due to vertical dead loads.

It was assumed that the vertical dead load was equal to the weight of the overlying material plus the weight of the portion of the arch considered and also that this load was concentrated at the panel points. The following are the dead loads used.

Panel Point	9 1	8 2	7 3	6 4	5
Concrete	17760	10510	8260	7180	6860
Earth	21150	14130	9460	6860	6040
Total Load	38910	24640	17720	14040	12900

By the use of table 3, for the abutment, for

W on 1, M = - 1.452 x 38910 x 50 = - 2724870 inch pounds

" " 2, M = - 1.536 x 24640 x 50 = - 1892350 " "

W on 3, M = - 0.876 x 17720 x 50 = - 776140 inch pounds

" " 4, M = 0 ------------------ = 0 " "

" " 5, M = + 0.744 x 12900 x 50 = + 479880 " "

" " 6, M = + 1.152 x 14040 x 50 = + 808700 " "

" " 7, M = + 1.140 x 17720 x 50 = + 1010040 " "

" " 8, M = + 0.768 x 24640 x 50 = + 946170 " "

" " 9, M = + 0.264 x 38910 x 50 = + 513610 " "

Totals = - 5393360, + 3758400.

Resultant = - 1634960 inch-pounds.

Similarly for each of the other panel points the following resultant bending moments were found:-

	10	9	8	7	6	
Panel Point	0	1	2	3	4	5
Resultant M	-1634960	+2719620	+1069080	+367270	-329620	-704700

Since the dead load is constantly applied the resultant moments will be the sum of the positive and negative moments.

<u>Moments due to horizontal live loads.</u>

The unit horizontal live load was assumed to be one third of the unit vertical live load. This load was considered to be distributed over a surface of the same depth as the vertical projection of the upper surface of the panel sections of the arch rib. The total horizontal load for each panel was considered to be concentrated at the panel points as follows:-

	9	8	7	6	
Panel Point	1	2	3	4	5
Hor. Live Load	1770	1150	620	200	0

The horizontal loads were placed on the points which, when loaded with vertical live loads, produce a maximum positive or a maximum negative moment at the section considered and the bending moments due to these horizontal live loads were computed using table 4. For the

abutment,

H on 6, M = + 1.57 x 200 x 50 = + 15500 inch pounds

" " 7, M = + 1.58 x 620 x 50 = + 48670 " "

" " 8, M = + 1.16 x 1150 x 50 = +66550 " "

" " 9, M = + 0.48 x 1770 x 50 = +42480 " "

Total = + 173200 inch pounds

H on 1, M = - 2.71 x 1770 x 50 = - 239830 inch pounds

" " 2, M = - 2.83 x 1150 x 50 = - 162370 " "

" " 3, M = - 2.21 x 620 x 50 = - 68510 " "

" " 4, M = - 1.66 x 200 x 50 = - 16290 " "

Total = - 487000 inch pounds

Similarly for each of the other panel points the following bending moments were found:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
M (+)	+173200	+137010	+191830	+130340	+ 25600	- 3340
M (-)	-487000	- 20890	- 41500	- 93960	-102910	-116620

<u>Moments due to horizontal dead loads</u>

The unit horizontal dead load was assumed to be one third of the unit vertical dead load, distributed over a surface of the same depth as the vertical projection of the upper surface of the panel sections of the arch rib. The total horizontal dead loads were considered to be concentrated at the panel points as follows:-

Panel Point	9 1	8 2	7 3	6 4	5
Hor. Dead Load	2400	1220	570	170	0

Then, since the dead load is constantly applied, by the use of table 4, for the abutment for

H on 1, M = - 2.71 x 2400 x 50 = - 325200 inch-pounds

" " 2, M = - 2.83 x 1220 x 50 = - 172630 " "

H on 3, M = - 2.21 X 570 x 50 = - 62980 inch-pounds

" " 4, M = - 1.66 x 170 x 50 = - 14110 " "

" " 5, M = 0 ---------------- = 0 " "

" " 6, M = + 1.57 x 170 x 50 = + 13430 " "

" " 7, M = + 1.58 x 570 x 50 = + 44740 " "

" " 8, M = + 1.16 x 1220 x 50 = + 70760 " "

" " 9, M = + 0.48 x 2400 x 50 = + 57600 " "

Totals = - 574920, + 186530 inchpounds

Resultant = - 388390 inchpounds

Similarly for each of the other panel points the following resultant bending moments were found:-

	10	9	8	7	6	
Panel Point	0	1	2	3	4	5
Resultant M	-388390	+154920	+170300	+ 34040	- 89400	-134530

Temperature Stresses

The arch expands or contracts with a change in temperature thus causing a negative thrust and positive bending moment with a fall in temperature and a positive thrust and negative bending moment with a rise in temperature.

Temperature moments

The moments were figured for a fall in temperature from 70° F to 0° F or 70° F in all.

If t = extreme temperature change = 70° F.,

e = coefficient of expansion of concrete = .0000055 per 1° F,

E = modulus of elasticity of concrete = 3,000,000 lbs. sq. in.,

I = moment of inertia of section of arch at crown in inches^3,

H_t = horizontal thrust in pounds due to change of temperature,

M_t = bending moment in inch pounds due to change of temperature,

k = rise of arch in feet,

then $H_t = \frac{45}{4}\frac{teEI}{144k^2}$ -(13)

(crown) $M_t = \frac{15}{4}\frac{teEI}{12k}$ -(14)

(Springing line) $M_t = \frac{15}{2}\frac{teEI}{12k} = 2\ M_t$ (crown) - - - - - - - - - - (15)

Substituting :- k = 50 ft., t = 70° F, e = .0000055, E = 3000000,

$I = \frac{bd^3}{12} = 157464$ inches,

M_t (crown) = 1136690 inch pounds

M_t (Springing line) = 2273380 inch pounds

H_t = - 5680 pounds.

Also the temperature bending moments at the different panel points bear the following ratios to that at the crown and the respective bending moments are as follows:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Ratio	2.00	0.92	0.08	0.52	0.88	1.00
M_t	+2273380	+1045750	+90930	+591080	+1000290	+1136690

Temperature shear

The vertical shear due to a change of temperature may be considered as equal to that of a uniform load which produces the horizontal thrust H_t, the equilibrium polygon of which is a parabola and the shear diagram similar to that of a beam uniformly loaded. This shear is zero at the crown and increases to a maximum at each abutment.

To find the panel load, W, which produces this value of H for an arch of ten panels, the sum of the coefficients for H, table 1, for loads on all panel points = 2.4997. Then

$W = \frac{H_t k}{2.4997c} = 2270$ pounds

Thus the shear diagram may be drawn

Point	10 0	9 1	8 2	7 3	6 4	5

Shear	10220	7940	5680	3400	1135	-1135

Horizontal thrusts.

The stresses at any point produced by both thrust and bending moment are usually the greatest when due to the maximum bending moment at the point and the total horizontal thrust from the loads which have produced that maximum bending moment.

Thus, referring to table 1, for vertical live loads, the horizontal thrust at the abutment

$H_L = (0.4320 + 0.3308 + 0.1920 + 0.0607)\ 7750 = 7840$ pounds.

Similarly, for each of the other panel points the following horizontal thrusts were found:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
H_L (+)	+7840	+8340	+4520	+8640	+11030	+10340
H_L (-)	+7840	+11030	+14850	+11500	+8340	+9040

Similarly for vertical dead loads the following horizontal thrusts were found:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
H_D	+44080	+44080	+44080	+44080	+44080	+44080

Referring to table 2, for horizontal live loads, the horizontal thrust at the abutment, for

H on 6, H_L = +0.490 x 200 = + 100 pounds

" " 7, H_L = +0.428 x 620 = + 200 "

" " 8, H_L = +0.289 x 1150 = + 330 "

" " 9, H_L = +0.106 x 1770 = + 190 "

Total = + 880 pounds

H on 1, H_L = -0.894 x 1770 = -1580 pounds

" " 2, H_L = -0.711 x 1150 = - 820 "

" " 3, H_L = -0.572 x 620 = - 350 "

H on 4, H_L = -0.510 x 200 = - 100 pounds

Total = -2850 pounds.

Similarly for each of the other panel points the following horizontal thrusts were found:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
H_L (+)	+880	+1070	+160	+680	+690	+190
H_L (-)	-2850	-1270	+780	+680	+1070	+1560

Similarly also for horizontal dead loads the following horizontal thrusts were found:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Resultant H_D	-2520	-120	+1100	+1670	+1840	+1840

The horizontal thrust due to temperature as found on page 10 = - 5680 pounds.

Results

Collecting the different bending moments and horizontal thrusts found above and reducing them to unit stresses by the formula S = Mc/I for bending moments and the formula S = P/A for horizontal thrusts, the following results were obtained:-

Panel Point	10 0	9 1	8 2	7 3	6 4	5
I (inches)	3692260	577600	316650	218600	168200	157460
c (inches)	77.28	41.64	34.08	30.12	27.60	27.00
Area section (sq. in.)	1854.72	999.36	817.92	722.88	662.40	648.00

Total Bending Moments Due To

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Vert. live l'd (+)	+157630	+47890	+74540	+94860	+72540	+58590
(-)	-149730	-39990	-68350	-87420	-64630	-50220

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Vert.dead l'd	-1634960	+2719620	+1069080	+367270	-329620	-704700
Hor. live l'd(+)	+173200	+137010	+191830	+130340	+25600	-3340
(-)	-487000	-20890	-41500	-93960	-102910	-116620
Hor. dead l'd	-388390	+154920	+170300	+34040	-89400	-134530
Temperature	+2273380	+1045750	+90930	+591080	+1000290	+1136690

Total Horizontal Thrusts Due To

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Vert.live l'd(+)	+7840	+8340	+4520	+8640	+11030	+10340
(-)	+7840	+11030	+14850	+11500	+8340	+9040
Vert.dead l'd	+44080	+44080	+44080	+44080	+44080	+44080
Hor.live l'd (+)	+880	+1070	+160	+680	+690	+190
(-)	-2850	-1270	+780	+880	+1070	+1560
Hor.dead l'd	-2520	-120	+1100	+1670	+1840	+1840
Temperature	-5680	-5680	-5680	-5680	-5680	-5680

Unit Stresses Due To Bending Moments Due To

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Vert.live l'd(+)	+3.30	+3.45	+8.02	+13.38	+12.46	+10.04
(-)	-3.13	-2.88	-7.36	-12.04	-10.60	-8.61
Vert.dead l'd	-34.22	+196.06	+115.06	+50.60	-54.08	-120.84
Hor.live l'd (+)	+3.62	+9.88	+20.64	+17.96	+4.20	-0.57
(-)	-10.19	-1.51	-4.46	-12.94	-16.88	-19.99
Hor.dead l'd	-8.12	+11.16	+18.32	+4.69	-15.01	-23.06
Temperature	+47.58	+75.39	+9.78	+81.44	+164.14	+195.32

Unit Stresses Due To Horizontal Thrusts Due To

Panel Point	10 0	9 1	8 2	7 3	6 4	5
Vert.live l'd(+)	+4.23	+8.35	+5.52	+11.94	+16.65	+15.96

Panel Point		10 0	9 1	8 2	7 3	6 4	5
	(-)	+4.23	+11.04	+18.15	+15.91	+12.59	+13.95
Vert.dead l'd		+23.77	+44.11	+53.89	+60.97	+66.55	+68.02
Hor.live l'd	(+)	+0.47	+1.07	+0.19	+0.94	+1.04	+0.29
	(-)	-1.54	-1.27	+0.95	+1.22	+1.62	+2.41
Hor.dead l'd		-1.36	-0.01	+1.17	+2.31	+2.78	+2.84
Temperature		-3.06	-5.68	-6.94	-7.85	-8.58	-8.77
Total unit Stresses	(+)	+39.27	+343.78	+225.65	+236.38	+200.25	+139.23
	(-)	+15.32	+326.41	+198.56	+184.31	+142.53	+101.27

CONCLUSION.

These stresses are within the allowable limits usually given; Therefore it would seem that the arch is well designed.

Table 1.

$n = b/c$	0	2	4	6	8	
y_0 = -	1.2	1.2	1.2	1.2	1.2	times k
y_1 = -----	0.1333	0.2222	0.2857	0.3333	0.3704	times k
y_2 = ----	0.1333	0	-0.2222	-0.6667	-2.0	times k
P_1 = -----	0.5	0.648	0.784	0.896	0.972	times W
P_2 = ----	0.5	0.352	0.216	0.104	0.028	times W
H = -----	0.4687	0.4320	0.3308	0.1920	0.0607	times c/k W

Table 2.

$n = b/c$	0	2	4	6	8	
x_0 = ----	0.000	0.016	0.128	0.432	1.024	times c
x_1 = ----	0.33	0.40	0.69	1.53	4.60	times c
x_2 = ----	0.33	0.38	0.49	0.63	0.81	times c
P = ----	0.375	0.346	0.265	0.154	0.049	times k/c·H
H_1 =-----	0.500	0.510	0.572	0.711	0.894	times H
H_2 =-----	0.500	0.490	0.428	0.289	0.106	times H

Table 3
Bending Moments Due to Vertical Loads.

	Abut.					Crown	
	0	1	2	3	4	5	
W on 9-----	+0.264	+0.072	-0.060	-0.144	-0.156	-0.120	times cW
W on 8 ----	+0.768	+0.192	-0.204	-0.420	-0.444	-0.288	times cW
W on 7-----	+1.140	+2.228	-0.372	-0.648	-0.600	-0.240	times cW
W on 6-----	+1.152	+0.132	-0.480	-0.672	-0.444	+0.192	times cW
W on 5-----	+0.744	-0.072	-0.444	-0.372	+0.144	+1.128	times cW
W on 4-----	0.	-0.312	-0.204	+0.312	+1.248	+0.192	times cW
W on 3-----	-0.876	-0.432	+0.336	+1.428	+0.432	-0.240	times cW
W on 2-----	-1.536	-0.216	+1.284	+0.576	+0.048	-0.288	times cW
Sum--------	-3.864	-1.032	-1.764	-2.256	-1.668	-1.296	times cW
Sum--------	+4.068	+1.236	+1.956	+2.448	+1.872	+1.512	times cW

					Abut.	
	6	7	8	9	10	
W on 9-----	-0.024	+0.132	+0.336	+0.612	-1.452	times cW
W on 8-----	+0.048	+0.576	+1.284	-0.216	-1.536	times cW
W on 7-----	+0.432	+1.428	+0.336	-0.432	-0.876	times cW
W on 6-----	+1.248	+0.312	-0.204	-0.312	0.	times cW
W on 5-----	+0.144	-0.372	-0.444	-0.072	+0.744	times cW
W on 4-----	-0.444	-0.672	-0.480	+0.132	+1.152	times cW
W on 3-----	-0.600	-0.648	-0.372	+0.228	+1.140	times cW
W on 2-----	-0.444	-0.420	-0.204	+0.192	+0.768	times cW
W on 1-----	-0.156	-0.144	-0.060	+0.072	+0.264	times cW
Sum- ------	-1.668	-2.256	-1.764	-1.032	-3.864	times cW
Sum--------	+1.872	+2.448	+1.956	+1.236	+4.068	times cW

Table 4.

Bending Moments Due To Horizontal Loads

(Coefficients are to be multiplied by kH)

H on	Abut. 0	1	2	3	4	Crown 5	6	7	8	9	Abut. 10
9	+0.48	+0.13	-0.11	-0.25	-0.29	-0.22	-0.05	+0.22	+0.60	+1.08	-2.71
8	+1.16	+0.29	-0.31	-0.63	-0.68	-0.45	+0.06	+0.85	+1.91	-0.12	-2.83
7	+1.57	+0.34	-0.47	-0.87	-0.85	-0.42	+0.43	+1.69	+0.94	-0.36	-2.21
6	+1.58	+0.28	-0.54	-0.89	-0.76	-0.17	+0.90	+0.99	+0.60	-0.29	-1.66
5	The bending moment at each point is neutralized by loads applied to both the right and left of the crown, which is the only manner in which they can occur.										
4	-1.66	-0.29	+0.60	+0.99	+0.90	-0.17	-0.76	-0.89	-0.54	+0.28	+1.58
3	-2.21	-0.36	+0.94	+1.69	+0.43	-0.42	-0.85	-0.87	-0.47	+0.34	+1.57
2	-2.83	-0.12	+1.91	+0.85	+0.06	-0.45	-0.68	-0.63	-0.31	+0.29	+1.16
1	-2.71	+1.08	+0.60	+0.22	-0.05	-0.22	-0.29	-0.25	-0.11	+0.13	+0.48

Table 5. (see fig.5)

$A = E \cos^{n=1}\phi\, d^3$ Not Constant. n For $A = E\cos^{n}\phi\, d^3$ Const.

Joint	Cot. ϕ	ϕ	d	d^3	Cos ϕ	Cos ϕd^3	n	a	a^2
1	6.608	8°36'1/4	4.55	94.20	0.998	94.01	3.56	0.66	0.4356
2	3.181	17°27'	4.80	110.59	0.954	105.50	2.78	0.56	0.3136
3	2.599	21°02'1/2	4.90	117.65	0.933	109.77	3.92	0.27	0.0729
4	2.191	24°31'1/2	5.07	130.32	0.909	118.46	2.82	0.24	0.0576
5	1.887	27°55'	5.25	144.70	0.883	127.77	4.53	0.33	0.1089
6	1.663	31°01'	5.45	161.88	0.857	138.73	3.02	0.75	0.5625
7	1.456	34°29'	5.67	182.28	0.824	150.20	2.93	0.47	0.2209
8	1.259	38°27	5.96	211.71	0.783	165.77	2.51	0.09	0.0081
9	1.035	44°01'	6.40	262.14	0.719	188.48	2.79	0.24	0.0576
10	0.849	49°40'	7.06	351.90	0.647	227.68	3.35	0.30	0.0900

Table 5. (see fig.5)

Joint	Cot.ϕ	ϕ	d	d^3	Cos ϕ	Cosϕd^3	n	a	a^2
11	0.656	56°44'	8.50	614.12	0.548	336.54	3.60	0.48	0.2304
12	0.493	63°45'1/4	11.00	1331.00	0.442	588.30			
							35.81	$\Sigma a^2 =$	2.1581

mean 3.26

a = n - mean.

$n = 3.26 \pm .6745 \sqrt{\frac{2.1581}{10}}$

$= 3.26 \pm .31$

$A = E\theta\cos\phi$

θ = moment of Inertia of section of

ϕ = angle from crown to joint.

E = coefficient of elasticity of material.

$A = E \cos\phi^{3.26 \pm .31} d^3 = k$

A = k = constant

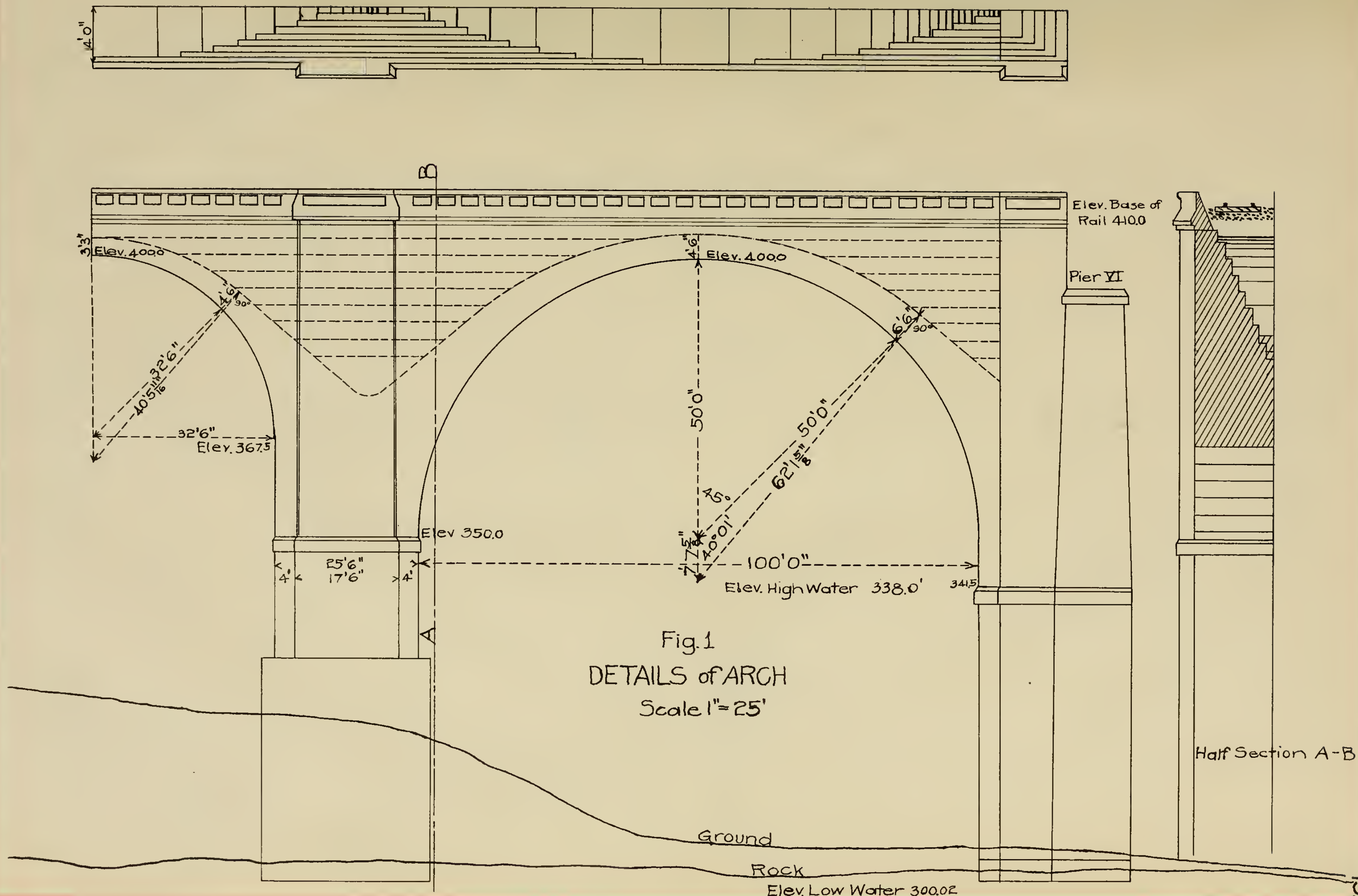

Fig. 1

DETAILS of ARCH

Scale 1" = 25'

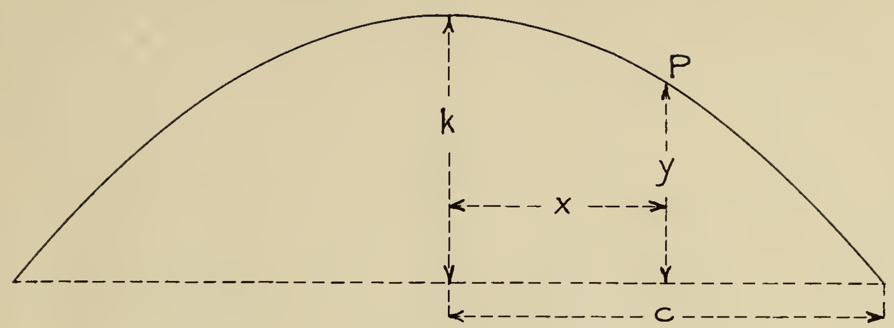

Fig. 2

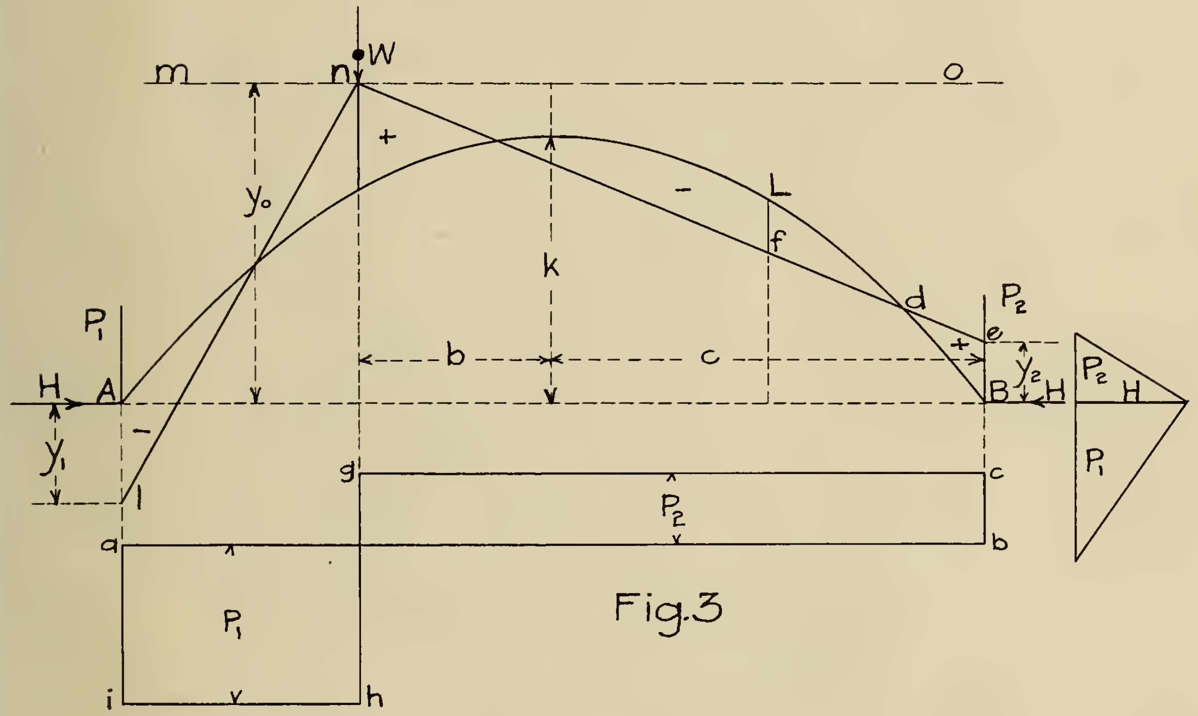

Fig. 3

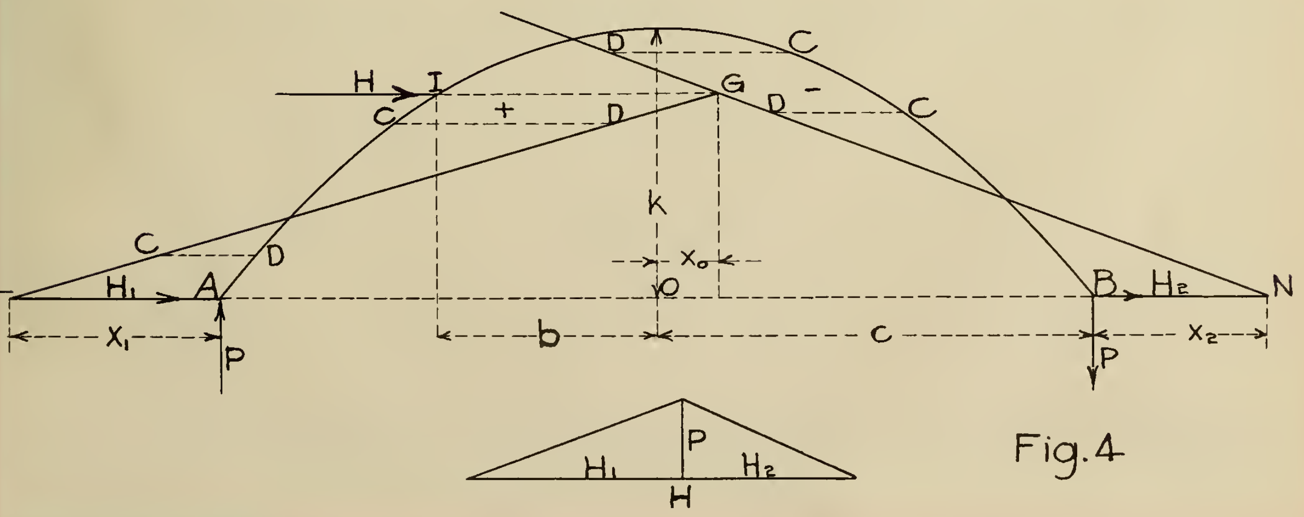

Fig. 4

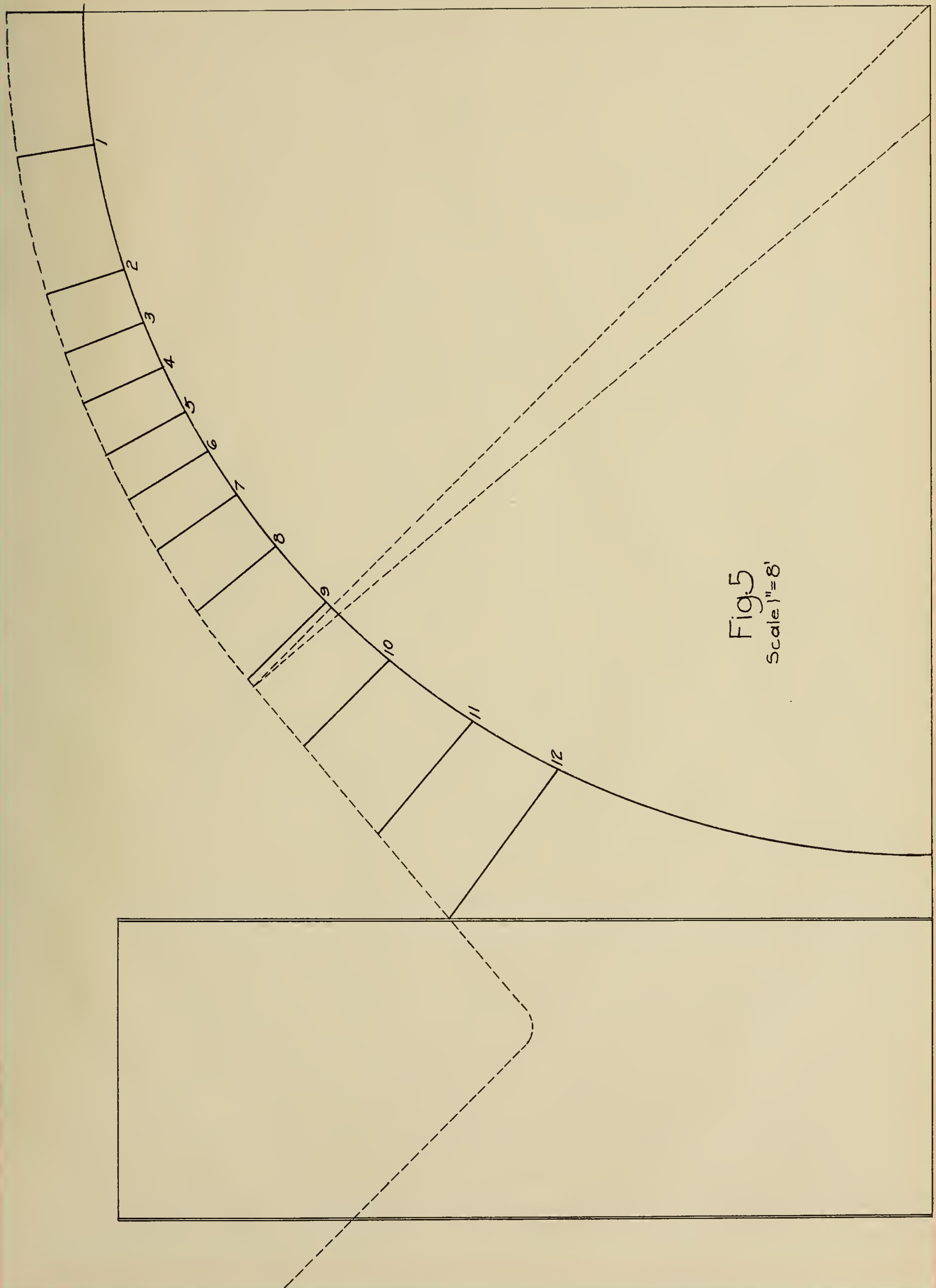

Fig. 5
Scale 1" = 8'